THE ORIGIN OF SPIN: A CONSIDERATION OF TORQUE AND CORIOLIS FORCES IN EINSTEIN'S FIELD EQUATIONS AND GRAND UNIFICATION THEORY

N. Haramein[¶] and E.A. Rauscher[§]
[¶]The Resonance Project Foundation, haramein@theresonanceproject.org
[§]Tecnic Research Laboratory, 3500 S. Tomahawk Rd., Bldg. 188, Apache Junction, AZ 85219 USA

Received January 1, 2004

Abstract. We address the nature of torque and the Coriolis forces as dynamic properties of the spacetime metric and the stress-energy tensor. The inclusion of torque and Coriolis effects in Einstein's field equations may lead to significant advancements in describing novae and supernovae structures, galactic formations, their center super-massive black holes, polar jets, accretion disks, spiral arms, galactic halo formations and advancements in unification theory as demonstrated in section five. We formulate these additional torque and Coriolis forces terms to amend Einstein's field equations and solve for a modified Kerr-Newman metric. Lorentz invariance conditions are reconciled by utilizing a modified metrical space, which is not the usual Minkowski space, but the U_4 space. This space is a consequence of the Coriolis force acting as a secondary effect generated from the torque terms. The equivalence principle is preserved using an unsymmetric affine connection. Further, the U_1 Weyl gauge is associated with the electromagnetic field, where the U_4 space is four copies of U_1. Thus, the form of metric generates the dual torus as two copies of U_1 x U_1, which we demonstrate through the S^3 spherical space, is related to the SU_2 group and other Lie groups. Hence, the S^4 octahedral group and the cuboctahedron group of the GUT (Grand Unification Theory) may be related to our U_4 space in which we formulate solutions to Einstein's field equations with the inclusion of torque and Coriolis forces.

1. INTRODUCTION

Current standard theory assumes spin/rotation to be the result of an initial impulse generated in the Big Bang conserved over billions of years of evolution in a frictionless environment. Although this first theoretical approximation may have been adequate to bring us to our current advanced theoretical models, the necessity to better describe the origin and evolution of spin/rotation, in an environment now observed to have various plasma viscosity densities and high field interaction dynamics which is inconsistent with a frictionless ideal environment, may be paramount to a complete theoretical model. We do so by formulating torque and Coriolis forces into Einstein's field equations and developing a modified Kerr-Newman solution where the spacetime torque, Coriolis effect and torsion of the manifold becomes the source of spin/rotation. Thus, incorporating torque in Einstein's stress energy term may lead to a more comprehensive description of the dynamic rotational structures of organized matter in the universe such as galactic formations, polar jets, accretion disks, spiral arms, and galactic halos without the need to resort to dark matter/dark energy constructs. These additions to Einsteinian spacetime may as well help describe atomic and subatomic particle interactions and produce a unification of fundamental forces as preliminarily described in section five of this paper.

Modification of the field equations with the inclusion of torque requires an unsymmetric affine connection to preserve the Principle of Equivalence and inhomogeneous Lorentz invariance, which includes translational invariance as well as rotational invariance and, hence, spin. The antisymmetric torsion term in the stress-energy tensor accommodates gauge invariance and maintains field transformations. Although the affine connection is not always a tensor, its antisymmetric components relate to torsion as a tensor. This is the case because when only the unsymmetric part is taken, the affine connections no longer disallows the existence of the tensor terms. We demonstrate that such new terms lead to an intrinsic spin density of matter which results from torque and gyroscopic effects in spacetime. The conditions on the Riemannian geometry in Einstein's field equations and solutions are also modified for torque and Coriolis forces and spacetime torsion condition. The torque and torsion terms are coupled algebraically to stress-energy tensor. The effect of the torque term leads to secondary effects of the Coriolis forces that are expressed in the metric. Torsion is a state of stress set up in a system by twisting from applying torque. Hence, torque acts as a force and torsion as a geometric deformation. The gauge conditions for a *rotational* gauge potential, $\Gamma^{\beta}_{\alpha\sigma}$ are used.

R. L. Amoroso, B. Lehnert & J-P Vigier (eds.) Beyond The Standard Model: Searching For Unity In Physics, 153-168,

The affine connection relates to transformations as translations and rotations in a uniform manner and represents the plasticity of the metric tensor in general relativity. Connections can carry straight lines into straight lines and not into parallel lines, but they may alter the distance between points and angles between lines. The affine connection $\Gamma^{\sigma}_{\mu\nu}$ has 64 components or 4^3 components of A_4. Each index can take on one of four values yielding 64 components. The symmetric part of $\Gamma^{\sigma}_{\mu\nu}$ has 40 independent components where the two symmetric indices give ten components including the times four for the third index. The torsion tensor $T_{\mu\nu\sigma}$ has 24 independent components and it is antisymmetric in the first two indices, which gives us six independent components and four independent components for the third index (indices run 1 to 4). These independent components relate to dimensions in analogy to the sixteen components of the metric tensor $g_{\mu\nu}$. If this tensor is symmetric then it has ten independent components. Note for a trace zero, *tr* 0 symmetric tensor, we have six independent components. The components of a tensor are, hence, related to dimensionality.

It appears that the only method to formulate the modified Einstein's equations, to include torque and Coriolis terms, is to utilize the U_4 spacetime and not the usual four-dimensional Minkowski space, M_4. This is the case because the vectors of the space in spherical topology have directionality generating a discontinuity or *part in the hairs* of a sphere whereas a torus topology can have its vectors curl around its short axis having *no parts in the hairs* so that no discontinuity of the vector space exists. Thus all the vectors of the space obey invariance conditions. Also, absolute parallelism is maintained. The U_4 space appears to be the only representation in which we can express torsion, resulting from torque, in terms of the Christoffel covariant derivative, which is used in place of the full affine connections where ∇_ρ represents the covariant derivative in U_4 spacetime using the full unsymmetric connections. Thus we are able to construct a complete, self-consistent theory of gravitation with dynamic torque terms and which results in modified curvature conditions from metrical effects from torsion. In the vacuum case, we assume $\delta\int R\, d^4x = 0$ where R is denoted as the scalar curvature density in U_4 spacetime. This new approach to the affine connection may allow the preservations of the equivalence principle. The usual nonsymmetric stress-energy tensor is combined with its antisymmetric torque tensor. The U_4 is key to the structure of matter affected by the structure of spacetime. We present in detail the manner in which the U_4 group space relates to the unification of the four force fields. The structure of U_4 is four copies of U_1, the Weyl group, as $U_4 = U_1 \times U_1 \times U_1 \times U_1$ where U_1 x U_1 represents the torus. Hence U_4 represents the dual torus structure. In this case we believe the U_4 spacetime, which allows a domain of action of torque and Coriolis effects, is a model of the manner in which dynamical properties of matter-energy arise.

Further, in section five we show that the 24 elements of the torsion tensor can be related to the 24 element octahedral gauge group S^4 which are inscribed in S^2, and that the 24 element octahedral gauge is related to the cube through its being inscribed in S^2. The 24 element group through S^2 yields the cuboctahedral group which we can relate to the U_4 space; thus, we can demonstrate a direct relationship between GUT theory to Einstein's field equations in which a torque tensor and a Coriolis effect is developed and incorporated.

2. ANALYSIS OF TORQUE AND CORIOLIS FORCES

In this section we present some of the fundamental descriptions of the properties of the torque and Coriolis forces. We examine the forces, which appear to yield a picture of galactic, nebula, and supernova formation. We apply these concepts to Einstein's field equations and their solutions. The angular momentum is $L = \hbar$ and $\underline{L} = \underline{r} \times \underline{p}$ where $\underline{r}$ is a radial variable and $\underline{p}$ is a linear momentum. The torque

(1) $$\underline{\tau} = \frac{d\underline{L}}{dt} = \underline{r} \times \underline{F}$$

where $\underline{F}$ is force and the conservation theorem for the angular momentum of a particle states that if the total torque $\underline{\tau}$ is zero then

(2) $$\dot{\underline{L}} = \frac{d\underline{L}}{dt} = 0$$

and thus the angular momentum is conserved. In the case where $\underline{\tau} \neq 0$ then $\underline{L}$ is not conserved. Torque is a twisting or turning action. Whereby

(3) $$\underline{\tau} = \underline{r} \times \underline{F} = \underline{r} \times \frac{d}{dt}\left(m\underline{v}\right) = \underline{r} \times \frac{d\,\underline{p}}{dt} = \underline{r} \times \dot{\underline{p}}$$

for $\underline{r}$ is a constant. The force $\underline{F}$ is orthogonal to $\underline{\tau}$, and $\underline{r}$ is not parallel to $\underline{F}$. The centrifugal term is then given as

(4) $$c = \omega^2 r_0 \cos\theta$$

where ω is the rotation of a spherical body, such as the earth's angular velocity or rotation and r_0 its radius and θ is the angle of latitude. The Coriolis term is proportional to $2\underline{\omega} \times \underline{v}$ and is responsible for the rotation of the plane of oscillation of a Foucault pendulum. This is a method whereby the Coriolis force can be detected and measured.

The key to the gyroscopic effect is that the rate of change in its angular momentum is always equal to the applied torque. The direction of change of a gyroscope, therefore, occurs only when a torque is applied. The torque is

(5) $$\underline{\tau} = \underline{r} \times \underline{F}\left(\frac{\pi}{2} - \gamma\right)$$

due to $\underline{F}$ which is perpendicular to $\underline{r}$ and L is the vector angular momentum $\underline{L} = \underline{r} \times \underline{p} = m\left(\underline{r} \times \underline{v}\right)$ where the vector $\underline{r}$ is taken along the axis of the gyroscope, and γ is a phase angle in the more general case.

A spinning system along an axis $\underline{r}$ with an angular momentum L has a torque in equation (1) when the force F is directed towards the center of gravity. If the total force, $\underline{F} = 0$ then $\dot{\underline{p}} = 0$ and linear momentum is conserved. Angular frequency, ω

(6) $$\omega^2 = \frac{1}{m}\left(\frac{d^2V}{dr^2}\right)$$

in the generalized case where $E = V + T$ where E is the total energy, V is the potential energy, T is kinetic energy and m is the mass of the system. A revolving of a particle has angular velocity

(7) $$\dot{\theta} = \frac{d\theta}{dt} = \frac{L}{mr^2}.$$

The rate of revolution decreases as r increases. If r = constant, then the areas swept out by the radius from the origin to the particle when it moves for a small angle $d\theta$, then

(8) $$dA = \frac{1}{2} r^2 d\theta$$

then $L = mr^2\dot{\theta}$ and has an area A. Then

(9) $$\frac{dA}{dt} = \frac{1}{2} r^2 \dot{\theta} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{L}{2m}$$

the radius vector $\underline{r}$ moves through $d\theta$ and for a central force, if the motion is periodic, for integration over a complete period t_0 of motion, we have the area of the orbit $A = \frac{Lt_0}{2m}$.

For a rigid uniform bar on a frictionless fulcrum, the moment of a force, or torque, in the simplest of mechanical terms, is the mass times the length of the arm. The product of the force and the perpendicular distance from the axis line of the action of the force is called the force arm or movement arm. The product of the force and its force arm is called the moment of the force or the torque τ. In more detail, we can describe torque in terms of a force couple exerted on the end of a rod for a solid or highly viscous material producing a twist displacement and hence shear stress and shear stain

(10) $$\frac{Shear\ stress}{Shear\ strain} = \frac{F/A}{\phi} = M$$

where F is the force, A is the area, ϕ is the angle of distortion and M is the shear modulus. Torsion is a state of stress set up in a system by twisting from an applied torque. Torque creates action or work. The external twisting effect is opposed by the shear stresses included in a solid or highly viscous material. That is, torsion is the angular strain produced by applying torque, which is a twisting force, to a body or system, which occurs when, for example, a rod or wire is fixed at one end (i.e., has an equal and opposite torque exerted on it) and rotated at the other. Therefore, torque is a force and torsion is a geometric deformation in the medium given by the torsion β

(11) $$\beta = \frac{\pi M r^4}{2d}$$

where r is the radius and d is the length or distance in flat space. The torque for such a system is defined by $\tau = \beta\ \Theta$ or

(12) $$\tau = \frac{\pi M r^4 \Theta}{2d}$$

where τ is in units of dyne-cm, M is the shear modulus and relates to the distortion of the shaft in dyne / cm^2 and Θ is the angle in radians through which one end of the shaft is twisted relative to the other. The moment of inertia is denoted as I and we substitute ω^2 from equation (6).

(13) $$E_k = \frac{1}{2} m v^2 = \frac{1}{2}\left(m r^2\right)\omega^2 = \frac{1}{2} I \omega^2 .$$

In our case, the term W for a generalized modulus in a medium that relates to the shear tensor of a fluid torsion (Ellis, 1971) is utilized. We employ a torque tensor as the $\lambda(m,E) = \tau_{\mu\nu}$ which is a term in Einstein's stress-energy tensor $\mathbf{T}_{\mu\nu}$ where torque is given as

(14) $$\tau_{\mu\nu} = \frac{\pi W r^4 \Theta_{\mu\nu}}{2R}$$

where R is the scalar curvature path in U_4 space over which torque acts and r is the radius of twist produced by the torquing force acting over R. In order to define the scalar sustained for maximum curvature, hence maximum torque in spacetime, we express the spatial gradient of R along the vector length R_ρ as $\nabla_\rho R = R_\rho$. This is the tensor form that can be utilized in Einstein's field equations. The distance or length is now denoted as R in a generalized curved space. We can denote R as $R_{\mu\mu}$. The quantity $\Theta_{\mu\nu}$ is a tensor in which rotation is included, and hence requires inhomogeneous Lorentz transformations and requires a modification of the topology of space from M_4 into U_4 space, which has intrinsic rotational components. In order to convert from Minkowski space to U_4 space we must define the relationship of the metric tensor and the coordinates for each space. We have the usual Minkowski metric $ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$ and the metric of U_4 space is given as $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$. We relate the metrics of the M_4 space and U_4 space as $\eta_{\mu\nu} = \frac{\partial x^\alpha}{\partial x^\mu}\frac{\partial x^\beta}{\partial x^\nu} g_{\alpha\beta}$. For any tensor $T_{\mu\nu}$ than $T_{\mu\nu} = \eta_{\mu\gamma} T_\nu^\gamma$ (all indices run 1 to 4). Then under the gauge transformation for an arbitrary $\lambda_{\mu\nu}$ as $\psi_{\mu\nu} \to \psi_{\mu\nu} + \lambda_{\mu\nu}$, we have $\int \sqrt{-\eta}\, d^4 x \tau^{\mu\nu} \lambda_{\mu\nu} = 0$ in U_4 space in analogy to $\int \sqrt{-g}\, d^4 x T^{\mu\nu}$ in Minkowski space.

Note that the spin field is the source of torsion and is the key to the manner in which spin exists in particle physics and astrophysics. The formulation of torque is not included in Einstein's field equations in any manner and is not incorporated in $R_{\mu\nu}, g_{\mu\nu}$ and $T_{\mu\nu}$ terms without modifications. Currently it appears that torque and Coriolis forces are eliminated by attaching the observer to a rotating reference frame and by assuming an absolute symmetry of the stress-energy tensor $T^{\mu\nu} = T^{\nu\mu}$ so to make the torque vanish [1]. We believe that inclusion of torque is essential to understanding the mechanics of spacetime, which may better explain cosmological structures and potentially the origin of rotation.

3. INCLUSION OF TORQUE AND CORIOLIS FORCE TERMS IN EINSTEIN'S FIELD EQUATIONS

In order to include torque, we must modify the original form of Einstein's field equations. The homogeneous and inhomogeneous Lorentz transformations involve linear translations and rotation, and hence angular momentum is accommodated. The time derivative of angular momentum, or torque, is not included in its field equations. Researchers have attempted to include torsion by different methods since Elie Cartan's letter to Einstein in the early 1930's [2]. However, we feel that an inclusion of torsion in Einstein's Field Equations demands a torque term to be present in the stress-energy tensor in order to have physical effects.

Two currently held key issues are addressed in which torque and Coriolis forces are eliminated. First, in reference [1] the complications of fractional differences are avoided by formulating them in terms of the size of spatial lower limit Planck length dimension, ℓ and the earth's gravitational acceleration g ~ 10^3 cm/sec^2. The choice of $g\ell << 1$ is made so that the accelerated frames undergo small accelerations which yields an approximately inertial frame. Black hole dynamical processes requires a relaxation $g\ell << 1$. If one considers a vacuum structure having a lattice form, then the conditions to include torque and Coriolis forces require a relaxation of the $g\ell << 1$ condition to be consistent with black hole physics and torque terms in relativity, then $g\ell \leq 1$ or $g\ell \sim 1$. Second, the torque and Coriolis forces are eliminated in a nonrelativistic manner by carefully choosing the observer's state of coordinates by preventing the latticework from rotating, i.e. by tying the frame of reference to a gyroscope that accelerates in such a manner that its centers of mass are chosen to eliminate these forces [1]. Hence, we have a major clue for including torque so as to fix our frame of reference to the fundamental lattice states, which includes rotation terms, and does not eliminate them. Then, for

$$\frac{d\underline{e}}{dt} = m(\underline{a} \cdot \underline{\theta}) - a(\underline{u} \cdot \underline{e}) \tag{15}$$

so that $a(\underline{u} \cdot \underline{e})$ is eliminated, noting that $\underline{u}$ is the four vector velocity and $\underline{e}$ is a basis vector in analogy to x, y, z. The incorrect transport equation is the Fermi-Walker transport equation because it is formulated in a rotating frame that eliminates torque. This equation acts at the center of mass so that I, the moment of inertia, is zero; hence this cannot be our reference frame.

It appears that we must utilize a different kind of rotational frame of reference. We have utilized this frame using the Kerr-Newman or Reissman-Nordstrom solutions with spin, as well as atomic spin and the spin of the whole universe as in our scaling law [3-8]. We thus generate a torus from our new basis vector set $\underline{e}$ [9].

Given these two conditions, we proceed to account for a torque term in Einstein's Field Equations. The angular momentum vector $\underline{L}$ for a system must change in order to have torque. Hence $\underline{L}$ is not orthogonal to $\underline{u}$, the four velocity; thus, a torque can be utilized in Einstein's field equations. Then

$$\frac{d\underline{L}}{dt} \neq (\underline{u} \times \underline{a}) \bullet \underline{L} \tag{16}$$

whereas in the Fermi-Walker transport case

$$\frac{d\underline{L}}{dt} = (\underline{u} \times \underline{a}) \bullet \underline{L} \tag{17}$$

where a is the four acceleration. The fact that a non-zero solution exists allows us to choose frames of reference that do not move with the system and include torque, which requires a variable acceleration. No longer is

$$\underline{L}^2 = \frac{3\hbar^2}{4} \tag{18}$$

constant because torque,

$$\underline{\tau} = \underline{\dot{L}} = \frac{d\underline{L}}{dt} \neq 0 \tag{19}$$

where $\underline{L}$ is the angular momentum.

Key to the inclusion of torque terms and its torsion effects is the modification of Einstein's field equations formulated in the generalized U_4 spacetime. This approach can be reconciled with conditions for affine connections and extended Lorentz invariance. Torsion resulting from torque is introduced as the antisymmetric part of the affine connection. The U_4 space appears to be the only spacetime metric that yields an unsymmetric affine connection and an antisymmetric torsion tensor term that preserves Lorentz Invariance [10,11]. We believe the U_4 spacetime allows a domain of action of torque and gives us a model of the manner in which dynamical properties of matter-energy arise out of the vacuum structure [12].

The vectors of the space in spherical topology have directionality (having a part in its hairs on a sphere) whereas a torus topology can have its vectors curl around its short axis (i.e., having no parts on the hairs of a torus) so that no discontinuity of the vector space exists. Thus all the vectors of the space obey invariance conditions. Also, absolute parallelism is maintained. Topologically, a torus is a surface of revolution generated by rotating a circle about a non-intersecting coplanar line as its axis.

For the vacuum gravitational field equations we introduce the antisymmetric torque term where $\tau^{\mu\nu,\sigma}_{;\sigma} = 0$ which gives us the antisymmetric derivative of a second-rank potential field $\psi_{\mu\nu,\sigma}$. Torsion appears to be the property of the geometry of spacetime, not the stress-energy tensor term; whereas torque is an inherent property of the stress-energy term. Thus torque and torsional effects on curvature can be expressed as tensor terms. We utilize the variational principle

(20) $$\delta \int \sqrt{-\eta}\,(R + L) d^4 x = 0$$

where R is subtended curvature density and L is the Lagrangian. We define $\eta_{\mu\nu}$ as $g_{\mu\nu}$ expressed in U_4 space. Then we can write the field equations

(21) $$-\eta^{\mu\nu} + \nabla_\sigma \tau^{\mu\nu\sigma} + 2\tau^{\mu}_{\sigma\beta}\tau^{\mu\sigma\beta} = 0$$

which are the gravitation and $\eta^{\mu\nu}$ is the Einsteinian tensor of U_4 space time. In vacuum $\sigma^{\mu\nu\sigma}_{;\sigma;\nu} = 0$ implies the existence of a conserved current, giving us a more generalized form of the variational principle or

(22) $$\delta \int (R_\rho + \kappa L_\rho + K L_\alpha) d^4 x = 0$$

for the source tensors

(23a) $$\delta L_\rho = T^{\mu\nu}_{\rho}\,\delta g_{\mu\nu}$$

and

(23b) $$\delta L_\alpha = j^{\mu\nu}_{\alpha}\,\delta\psi_{\mu\nu}$$

where $\mathbf{T}^{\mu\nu}_{\rho}$ is the density stress-energy tensor and L_ρ is the Lagrangian density. The constant κ is the coupling constant $\kappa = 8\pi$ and K is the coupling constant for torque term. We define

(24) $$J^{\mu\nu} = \kappa \mathrm{T}^{\mu\nu} - K j^{\mu\nu}/2$$

which the field equation

(25) $$\eta^{\mu\nu} - 2\tau^{\mu}_{\alpha\beta}\tau^{\nu\alpha\beta} = J^{\mu\nu}$$

which is given as the right side of the above equation (24) and $\tau^{\mu\nu}$ is the antisymmetric source term which arises from intrinsic spins where $T^{\mu\nu,\sigma}_{;\sigma} = -\dfrac{K}{2} J^{\mu\nu}$. Then gauge invariance implies $\int \sqrt{-\eta}\, d^4 x \tau^{\mu\nu} \lambda_{\mu\nu} = 0$ for an arbitrary gauge transformation $\lambda_{\mu\nu}$.

The stress-energy tensor can then be related to the stress tensor and the torque tensor as

(26) $$\tau^{\mu\nu,\sigma}_{;\sigma} = (3K/s\kappa)\tau^{\mu\nu}{}_{\alpha\beta}\, j^{\alpha\beta}\,.$$

In vacuum the static solution yields the line element

(27) $$ds^2 = e^{\nu} dt^2 - r^2 d\Omega^2 + e^{\lambda} dr^2$$

where λ and ν are functions of r only as $\lambda(r)$ and $\nu(r)$. The Ω term is an anharmonic object which preserves absolute parallelism. We can write a more generalized stress energy term as

(28) $$\mathbf{T}^{\mu\nu} = K \mathrm{T}^{\mu\nu} + \kappa \tau^{\mu\nu}$$

where the first term $\mathrm{T}^{\mu\nu}$ is the usual stress energy term where $\partial_\nu \mathrm{T}^{\nu}_{\mu} = 0$ and the second term $\tau^{\mu\nu}$ is the torque term and $\mathbf{T}^{\mu\nu}$ becomes the total stress energy term including torque. Note that both covariant and contravariant tensor notations are utilized. The most general form of Einstein's field equations with torque and the cosmological term, $\Lambda \neq 0$ in U_4 spacetime is

(29) $$R_{\mu\nu} - \frac{1}{2} R_{\mu\mu}\eta_{\mu\nu} - \Lambda\eta_{\mu\nu} = K\mathrm{T}_{\mu\nu} + \kappa\tau_{\mu\nu}$$

where we have the usual gravitation source terms $\mathrm{T}_{\mu\nu}$ and non-gravitational source terms $\tau_{\mu\nu}$ with Λ as the cosmological constant in U_4 space. Note that units of $c = G = 1$ are used in this section and that the cosmological constant in a torque field may yield correct approximations for the universal cosmological acceleration of distant objects.

A conceptual picture of the interpretation of Einstein's field equations is that the presence of matter-energy curves space and time. Torque is considered as a property of the stress-energy term, and the Coriolis forces are derived as secondary properties resulting from the torquing of matter-energy in spacetime. Hence, resulting Coriolis effects are driven by torquing on spacetime and therefore spacetime geometry is modified.

The Coriolis and centrifugal terms enter when we define a new frame of reference. We start from the Lorentz coordinates which holds everywhere

(30) $$\left(\frac{\partial}{\partial x^{\mu}}\right)\left(\frac{\partial}{\partial x^{\nu}}\right) = \eta_{\mu\nu} \, .$$

We define $\Gamma^{j}_{00} = \phi_{,j}$ for a given scalar potential field, ϕ for a Galilean rather than Lorentz coordinates. Then

(31) $$\left(\frac{\partial}{\partial x^{j}}\right)\left(\frac{\partial}{\partial x^{k}}\right) = \delta_{jk}$$

and $x^{0} = t$. The potential ϕ satisfies the Laplace-Poisson equation.

For rotation and translation, we have $x^{j} = A_{jk}x^{k} + a^{j}$ where the rotation matrix is $A_{j\ell}A_{k\ell} = \delta_{jk}$ and the translation part is given as a^{j}. Then $x^{k} = A_{jk}x^{j} - a^{k}$ for $a^{k} \equiv A_{jk}a^{j}$ which defines a new coordinate system. The

(32) $$\Gamma^{j}_{0k'} = \Gamma_{k'\ell} = \mathrm{A}_{j\ell}\mathrm{A}_{k'\ell}$$

produces the Coriolis forces from these transformations. From

(33) $$\Gamma^{j}_{00} = \frac{\partial\phi}{\partial x^{j}} + A_{jk}(\ddot{A}_{\ell k}x^{\ell} - \ddot{a}^{k})$$

gives us the centrifugal force, $A_{jk}\ddot{A}_{\ell k}$ and the inertial forces $\ddot{a}^{k}$ which are separated. Thus we have tensor notation which allows us to relate these terms to the stress-energy tensor of Einstein's field equations. The inertial forces $\ddot{a}^{k}$ is the second derivative with respect to time and $\dot{A}_{k'\ell}$ is the first time derivative.

The scalar potentials transform as $\phi = \phi - \ddot{a}^{k}x^{k} +$ additional higher order terms such as $a\left(\ddot{a}^{k}x^{k}\right)$ for Coriolis, $A_{j\ell}\dot{A}_{k'\ell}$ and centrifugal forces, $A_{jk}\ddot{A}_{\ell k}x^{\ell}$. If the additional higher order terms are zero, then no Coriolis and centrifugal terms are included. One can measure the quantity

$\frac{\partial\phi}{\partial x_{j}} = \Gamma^{j}_{00}$ but only in a finite range. We can express these terms in terms of the metric theory of gravity as

(34) $$\Gamma^{\alpha}_{\mu\nu} = g^{\alpha\beta}\,\frac{1}{2}\left(\frac{\partial g_{\beta\nu}}{\partial x^{\mu}} + \frac{\partial g_{\beta\mu}}{\partial x^{\nu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\beta}}\right) .$$

For

(35) $$\Gamma^{0}_{\alpha\beta} = -(\nabla_{\beta}dt \cdot \underline{e}_{\alpha}) \rightarrow 0$$

then the gradients $\nabla_\beta dt = 0$ for all β and $\nabla_\mu dt = 0$ for all velocity vectors $\dot{x}^k = \underline{v}$ and spatial vectors, x acting on arbitrary basis set, $\underline{e}_j$ or $\underline{e}_j \cdot \underline{e}_k = 0$. This is clearly not the case for centrifugal, torque and Coriolis terms. The gradient of proper universal time is not conveniently constant (as it is in the above case) when additional terms are included, hence we will need to redefine the geometric version of space and time by use of our vacuum equations, which we demonstrate in this section and in section 4 and relates to the U_4 metric. Hence, the key may be in relating the Gaussian curvature through a radius $a = \frac{1}{\boldsymbol{a}^2}$ to the cuboctahedron and dual torus form (see Fig. 1).

Even for an accelerated observer for a particle velocity $\underline{v} \equiv \frac{dx^j}{dx^0} \underline{e}_j$ then we have the inertial acceleration

(36) $$\frac{d^2 x^j}{d(x^0)^2} \underline{e}_j = -\underline{a} - 2\underline{\omega} \times \underline{v} + 2(\underline{a} \bullet \underline{v})\underline{v}$$

where $x^0 = t$ is the fourth component of space, which is time, and $2\underline{\omega} \times \underline{v}$ is the Coriolis term and $2(\underline{a} \bullet \underline{v})\underline{v}$ is the relativistic correction to an inertial frame. The signature we use is $(-,+,+,+)$. The expression in terms of the potential energy is $\omega^2 = \frac{1}{m}(\frac{d^2\phi}{dr^2})$ where ω is the angular velocity. This latter term requiring modification in order to include torque is

(37) $$\underline{\tau} = \frac{d\underline{L}}{dt} = \dot{\underline{L}}$$

where $\underline{\tau} = \underline{r} \times \underline{F}$, see equations (1),(2),(3).

Torque also has intrinsic properties of the spacetime manifold. One can relate the torsional effect as a geometrical effect on spacetime curvature topology in analogy to Riemannian geometry. Using the torque term from equation (14) which is in units of dyne-cm we return to our generalized stress-energy tensor

(38) $$\mathbf{T}^{\mu\nu} = \frac{8\pi c^4}{G} \mathrm{T}^{\mu\nu} + \frac{8\pi G}{c^4} \ell \tau^{\mu\nu}$$

where $\mathbf{T}^{\mu\nu}$ is the total stress-energy tensor including its torque term. The quantity in the usual stress energy and the new torque term includes the fundamental force [8]

(39) $$F = c^4 / G$$

in units of dynes. The units of the left side of the field equations are in cm^2, or length squared. The quantity ℓ is in cm and

(40) $$\ell = \left(\frac{G\hbar}{c^3}\right)^{1/2}$$

which is the Planck length and can be written as

(41) $$\ell = \left(\frac{c\hbar}{F}\right)^{1/2}$$

for the fundamental force in equation (39) . Now we can write the torque term as

(42) $$\frac{8\pi}{F}\left(\frac{c\hbar}{F}\right)^{1/2} \tau^{\mu\nu} = \frac{8\pi (c\hbar)^{1/2}}{F^{3/2}} \tau^{\mu\nu}.$$

Now we can write the total stress energy term as

(43) $$\mathbf{T}^{\mu\nu} = 8\pi F \mathrm{T}^{\mu\nu} + \frac{8\pi (c\hbar)^{1/2}}{F^{3/2}} \tau^{\mu\nu} = 8\pi F \left[\mathrm{T}^{\mu\nu} + \frac{(c\hbar)^{1/2}}{F^{5/2}} \tau^{\mu\nu} \right].$$

From equation (29) and (38) we can write our generalized field equations with the inclusion of torque as,

$$R_{\mu\nu} - \frac{1}{2} R_{\mu\mu} \eta_{\mu\nu} - \Lambda \eta_{\mu\nu} = \frac{8\pi c^4}{G} \mathrm{T}_{\mu\nu} + \frac{8\pi G}{c^4} \ell \tau_{\mu\nu} \tag{44}$$

where $\eta_{\mu\nu}$ represents the metric of tensor for the U_4 topological space. This topology is unique for the inclusion of the torque term in the stress-energy tensor in equation (44). Coriolis forces result from rotational effects of torque in this topology and also may yield a non-zero cosmological term, Λ, discussed the next section.

4. EXTENDED KERR-NEWMAN SOLUTION TO EINSTEIN'S FIELD EQUATIONS WITH THE INCLUSION OF TORQUE

We have developed a new solution to Einstein's field equations in the previous section which contains a torque term. This requires unsymmetric affine connections in the metrical space. To introduce torque into the Einstein-Maxwell equation, in order to unify gravity and electromagnetism, we must introduce an antisymmetric part into $F_{\mu\nu}$ divided by the number of permutations related to the degrees of freedom. We can then represent the simplest covariant second rank tensor potential to represent torsion, which we term $\psi_{\mu\nu,\sigma}$. The electromagnetic field vector is constructed from the vector fields as $F_{\mu\nu} = 2\phi_{\mu\nu}$ where $\phi_{\mu\nu}$ is the potentials. We define the torsion term in terms of generalized potentials as $\tau_{\mu\nu,\sigma} = \psi_{\mu\nu,\sigma}$. Gauge invariance is then expressed as $\psi_{\mu\nu} \to \psi_{\mu\nu} + \lambda_{\mu\nu}$ where $\lambda_{\mu\nu}$ is any vector field. Thus one expects that the second rank current density is conserved [13,14].

We proceed from the solutions to Einstein's field equations including the torque term conditions and determine that these conditions require the inclusion of the cosmological constant $\Lambda \neq 0$ and the modified stress-energy tensor. The Schwarzschild spacetime geometry for the Schwarzschild black hole gravitational field for a spherical coordinate line element, is given by

$$ds^2 = -(1-\frac{2M}{r})dt^2 + \frac{dr^2}{(1-2M/r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{45}$$

We consider the metric parameter, Φ for a non-zero cosmological constant of the form $\Lambda = -\frac{1}{2}\ell n(1-2M/r)$.

The normalization scale $\Phi = \frac{1}{2}\ell n(1-2M/r)$ for a frame of reference external to the black hole. We can also write this form of the cosmological constant as

$$e^{\Lambda} = \frac{1}{[1-2M/r]^{1/2}} \tag{46}$$

or

$$e^{-2\Lambda} = [1-2M/r] \tag{47}$$

at the Schwarzschild radius r_s , for a variable radius *M*(r). A slice through the equator of a spherical system and also between the two tori of a dual torus is given as

$$ds^2 = \frac{1}{[1-2m(r)/r]} dr^2 + r^2 d\phi^2 \tag{48}$$

which comprises an apparent flat space where $m(r) = M$. We can then write

$$ds^2 = e^{2\Lambda} dr^2 + r^2 d\phi^2 \tag{49}$$

for a non-zero cosmological constant Λ, for $r_s \propto \frac{M}{c^2}$, which is the Schwarzschild singularity. The global structure of the Schwarzschild geometry represents a method of embedding Feynman diagrams. The coordinate system that provides maximum insight into the Schwarzschild geometry is known generally as the Kruskal-Szekeres coordinate systems [15,16]. Charge and spin are relevant; for example, consider the Kerr-Newman or Reissner-Nordstrom generalization of the Schwarzschild geometry. For gravitational and electromagnetic fields, we solve the coupled Einstein-Maxwell field equations to include the constraints of M, mass, q, charge, and s , spin. The Kerr-Newman metric is written in the form

(50) $$ds^2 = -\frac{\Delta}{\rho^2}[dt - a\sin^2\theta d\phi]^2 + \frac{\sin^2\theta}{\rho^2}[(r^2 + a^2)d\phi - adt]^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 .$$

We define the quantities in terms of charge, q, and the quantity a is defined as $a \equiv s/M$, the angular momentum, which we usually define as L. This gives us a method to bring torque into our model since $\underline{\tau}$ is defined as $\underline{\tau} = \frac{d\underline{L}}{dt} = \dot{\underline{L}}$. Also torque is dependent on angular velocity $\dot{\Theta}$ which is expressed in terms of torque as $\tau = \frac{\pi W r^4 \Theta}{2R}$ where the angular velocity acceleration is $\dot{\Theta} = \frac{d\Theta}{dt} = \frac{L}{mr^2}$. Hence it appears we can expand the Kerr-Newman solution to accommodate torque. For the present the units of $c = G = 1$ are used. Using dimensional analysis we can consider the scalar magnitude of the torque, which is a vector. We can convert the units of torque into units proportional to cm^2. The units of torque are dyne-cm and the scalar part is $gm\frac{cm^2}{\sec^2} = ergs$.

Before proceeding further, we need to define two other quantities

(51) $$\rho^2 \equiv r^2 + a^2\cos^2\theta$$

and

(52) $$\Delta \equiv r^2 - 2mr + a^2 + q^2 .$$

Note we use the action integral $\int (R+\varepsilon)F_g d^4x$ so that we can convert mass in gm or density in gm/cm^3 into mass in cm or density in cm^{-2} by multiplying by 0.742 x 10^{-28} cm/gm and lengths in units $(3/8\pi P_0)^{1/2}$ and pressure in units ρ_0, mass in units $(3/32\pi\rho_0)^{1/2}$. Constraints on the Kerr-Newman geometric solution to Einstein's field equations give black hole topology for the condition $M^2 \geq q^2 + a^2$. Recall that the quantity, a contains spin and mass, in the condition where for M such that $M^2 \sim q^2 + a^2$. It is possible that, under imminent collapse, near r_s centrifugal forces and/or electrostatic and plasma electromagnetic repulsion will be delayed, or halt and collapse, and become balanced [17].

In the case of the Reissner-Nordstrom geometry which contains electromagnetic fields, for $q \neq 0$ but $s = 0$, spin is zero. The Kerr geometry is valid for an uncharged system or q = 0 and a Schwarzschild geometry for $q = s = 0$. The case we consider that is relevant to including torque is the case for $M^2 = q^2 + (s/M)^2$ for the Kerr-Newman geometry for a black hole rotation in the θ direction and spin along the z axis. Also, angular momentum will occur along the z axis only. For black holes q<<M (utilizing $G = c = 1$ units), the repulsive electrostatic force on protons of mass m_p is similar to the gravitational pull by a factor of

(53) $$\frac{electrostatic}{gravitational}\frac{force}{force} = \frac{eq}{m_\rho M} \sim \frac{e}{m_\rho} \sim 10^{20}$$

where M is the mass of the black holes.

We do not need to convert rectilinear coordinates x, y, z to the spherical coordinates r, θ, and ϕ. The θ coordinates moves or rotates in the x-y plane and ϕ moves in the zr plane where r is a radius vector from the origin of the x, y, z system. The spherical coordinate θ can go from $0 \le \theta \le 2\pi$ and $0 \le \phi \le \pi$ and $r \ge 0$. Then $x = r\sin\phi\cos\theta$ and $y = r\sin\phi\sin\theta$ and $z = r\cos\phi$. Also $r^2 = x^2 + y^2 + z^2$ and $\theta = \frac{y}{x}$ relating the variables x, y, z, and r and θ utilizing the Kerr-Newman extended solutions including torque in units of $c = \hbar = G = 1$ gives

$$ds^2 = \frac{-\Delta}{\rho^2}[dt - 2\sin^2\theta d\phi]^2 + \frac{\sin^2\theta}{\rho^2}[(r^2 + a^2)d\phi - adt]^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\pi\ell^2 W r^4 \Theta\cos^2\theta dr^2}{2mR} \tag{54}$$

where the latter term is the torque term $\tau = (\frac{\ell^2}{E})\frac{\pi W r^4 \Theta}{2R} = \frac{\pi\ell^2 W r^4 \Theta}{2mR}$ with precession defined as $\cos\theta$ and ℓ, *E* and *m* are in Planck units. Note that spin and torque are related. The Coriolis forces act as higher order terms which are smaller than the other terms but are still significant [17].

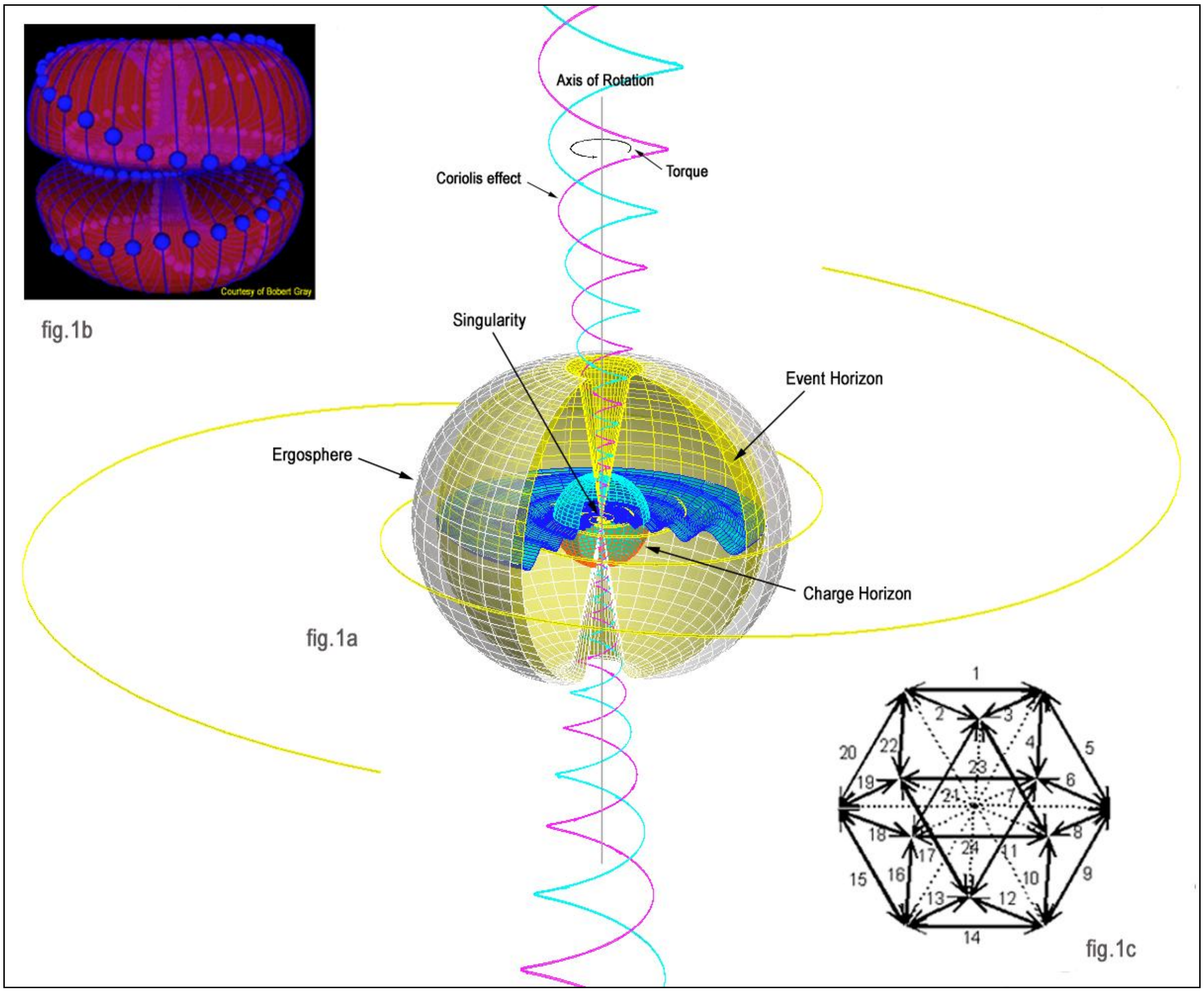

Figure 1. 1(a). is a topological representation of the Haramein-Rauscher solution as a result of the addition of torque and Coriolis force terms as an amendment to Einstein's field equations, which modifies the Kerr-Newman solution. The Lorentz invariance conditions are reconciled by utilizing a modified metrical space, which is not the usual Minkowski space, but the U_4 space. This space is a consequence of the Coriolis force acting as a secondary effect, which is generated from the torque term in the stress-energy tensor. In figure 1(b). Coriolis type dynamics of the dual U_1 x U_1 spacetime manifold are illustrated. The form of metric produces the dual torus as two copies of U_1 x U_1, which we demonstrate through the S^3 spherical space, is related to the SU_2 group and other Lie groups. In figure 1(c). the 24 element group through S^2 yields the cuboctahedral group which we can relate to the U_4 space (next section). Thus the S^4 octahedral group is related to the U_4 topology and we demonstrate that the cuboctahedral group relates to the GUT (Grand Unification Theory).

In this section we have shown that we can modify Einstein's field equations and the Kerr-Newman solution in order to accommodate torque and the Coriolis forces, which we term the Haramein-Rauscher solution. Since Einstein's field equations obey the Laplace-Poisson condition, the torquing of spacetime may be the result of the vacuum gradient density in the presence of matter-energy. Modification of the field equations makes it possible to include the torque terms and hence generate more realistic solutions. These solutions more comprehensively describe the dynamical rotational structures of galaxies, novae, supernovae, and other astrophysical structures which in this case are driven by a spacetime torque. Hence, with the inclusion of torque and Coriolis effects in Einstein's field equations, the spacetime manifold correlates well with the observable mechanisms of black holes, galactic topology, supernova formation, stellar plasma dynamics and planetary science such as ring formation and the Coriolis structure of atmospheric dynamics. This may lead to a model where the driving torque and the dynamical Coriolis forces of the spacetime manifold topology are responsible for the observed early formation of mature spiral galaxies [18]. Further, our model is consistent with galactic structures, the super-massive black hole at their centers, as well as polar jets, accretion disks, spiral arms and galactic halo formations.

5. THE GROUP THEORETICAL APPROACH TO A UNIFIED MODEL OF GRAVITATION INCLUDING TORQUE AND THE GUT THEORY

A test particle falling in a gravitational field accelerates relative to the observer's frame as

$$\frac{d^2\hat{x}^j}{dx_0^{\,2}}\underline{\hat{e}}^j = -\underline{a} - 2\underline{\omega}\times\underline{v} \tag{55}$$

where x_0 is the temporal component of $\mathrm{X} = (x, y, z, t)$ or $x_0 = t$ for $j = 0$ and in general j runs 0 to 3. The inertial acceleration of the observer's four space acceleration is $\underline{a}$. For the spatial vectors of the observer, $\underline{\hat{e}}_j$ are rotating with angular velocity, $\underline{\omega}$. In flat space this is the geodesic trajectory only if there is an additional rotational frame of reference $+2(\underline{a}\cdot\underline{v})\underline{v}$ [1]. This is not our case when we include the Coriolis effects.

We term $\underline{e}_0$ the points along the observer's path as its time direction $\underline{e}_0 = \underline{u} = \dfrac{dx_0}{d\tau}$ where τ is now defined as the proper time and the spatial components $\underline{\hat{e}}_j$ are the basis vectors. For tetrad orthogonality we have $\underline{\hat{e}}_i \cdot \underline{\hat{e}}_j = \delta_{ij}$, for Euclidian absolute parallelism or for the generators of Lorentz transformation, then the transport laws of a test particle space in curved spacetime appear as moving in a flat space. However, this is only a very limited approximation, as spacetime is curved and Riemannian in global space. The equivalence principle or the time rate of change of a vector occurs over finite distances, not just infinitesimal distances.

We define $\underline{a}$ the four acceleration $\underline{a} = \nabla_\mu \underline{u}$ and the angular velocity of rotation, of the spatial basis vectors, $\underline{e}_j$ in the Fermi-Walker transport theory, is $\underline{\omega}$. The Fermi-transport vectors are expressed relative to the inertial guidance gyroscope, $\underline{u}\cdot\underline{a} = \underline{u}\cdot\underline{\omega} = 0$. If $\underline{u}$ and $\underline{\omega}$ are zero then the parallel to the observer is $\nabla_u \underline{\hat{e}}_j = 0$. The proper time gives us the starting point of the geodesic with an affine parameter equal to the proper length. Hence, we see that the role of the Coriolis force, as well as including the torque term in Einstein's field equations, is again going to lead us to a U_4 space rather than an M_4 space, in which we utilize the inhomogeneous Lorentz transformation.

Further, we must consider a geometric picture in terms of finite group theory with finite generators and its relationship to the Lie group theory and their algebras having infinitesimal generators. These finite groups are the C_n groups. These groups can be related to the 24 element octahedral group, the $C[\overline{O}]$ and $C[O]$ groups. There is no real independent observer as the observer moves within the system since, in fact, under no circumstances could any observer be completely disconnected from the observed, since observation would then be impossible.

The affine connections that are utilized in general relativity also apply in crystallography. Under affine connections, transformations are linear and rotational in a uniform manner. Straight lines are carried in to straight lines and parallel lines, but distances between points and angles, lines can be altered. All representations of a compact group are discrete. Unitarity relates to the conservation of such quantities as energy, momentum, particle number, and other variables. The crystallographic lattice groups are finite groups: C_n and K_n specify the translations and rotations in a finite dimensional space. (Note in crystallography, the finite dimensional space involves arrays specifying elements of the groups in a spatial lattice.) This lattice structure appears to reflect the actual geometric structure of space and time [19]. The two torus satisfies conditions of a Lie group which can have an underlying manifold as a Lie algebra. This is necessary for the concept of invariance to exist. The McKay groups are a finite subgroup of the special unitary Lie groups such as SU_2, which is the set of unit determinants 2 x 2 complex matrices acting on C^2, the complex space. The SU_2 group is geometrically the 3 sphere, S^3 acting on C^2. Thus, we can relate its torus geometry to the Lie groups of the GUT scheme.

For the infinitesimal generators of the Lorentz group, we have an associated Lie algebra. However, if we have finite generators, we have a C_n group space. We might then say that M_4 space has a Lie algebra associated with it, whereas U_4 space has a finite C_n algebra associated with it. We might well expect this because of the group theoretical association of the double torus and the cuboctahedron, which is described by a crystallographic C_n group theory.

The Coriolis force comes from the metric; that is the spatial part or the left side of the field equations utilizing the double octahedral group or cuboctahedral geometry. For the U_4 metric we see that the $C[O]$ group naturally leads to the GUT scheme. Hence, the unification picture results directly from the geometry of spacetime with the consideration of finite group theory. The U_4 space directly relates the new Haramein-Rauscher equations of gravity, matter-energy, and torque with the GUT theories. Thus, we can construct a fundamental relation of cosmology and quantum particle physics by relating Lie groups and their infinitesimal generators of the Lie algebra, and groups having finite generators for finite groups.

The special unitary Lie groups, which are topological groups having infinitesimal elements of the Lie algebras, are utilized to represent the symmetry operations in particle physics and in infinitesimal Lorentz transformations. For example, the generators of the special unitary SU_2 group is composed of the three isospin operators, I as I_+, I_- and I_z having commutation relations $[I_+, I_-] = iI_z$. The generators of SU_3 are the three components of I, isospin, and hypercharge Y, and for other quantities which involve Y and electric charge Q. Thus, there are eight independent generators for the traceless 3 x 3 matrices of SU_3. The O_3^+ group of rotations is homomorphic to the SU_3 group.

The regular polyhedral groups, including the cube and the octahedron, form a complete set of finite subgroups of SO_3 the special orthogonal-3 group. The continuous Lie group SU_2 acts on a two dimensional real space in analogy to SO_3 acting on a three dimensional real space. Significantly, the S^3 group, also called the SU_2 group acts as a space which is the double cover of SO_3. That is SU_2 acts as a space that is a sphere, S^3, and SO_3 which is S^3 / $\{\pm 1\}$ so that SO_3 can be derived from its subgroup SU_2 by the plus and minus elements of SU_2 in order to form SO_3 [20-22]. The set of all rotations of a sphere is a useful example of a Lie group. They are a continuous infinity of rotations of an ordinary sphere or 2-sphere, S^2, which is embedded in SO_3. The rotations of S^2 form a 3-sphere modular plus or minus 1, called S^3 / $\{\pm 1\}$ which is embedded in SO_3. This group is the set of all special orthogonal 3x3 matrices. The finite subgroups of SO_3 are the symmetry groups of the various polyhedra which are inscribed on the sphere S^2 upon which SO_3 acts. These regular polyhedral groups are the symmetry groups for the five Platonic solids. The octahedron and icosahedron are inscribed in S^2, the symmetry group of 24 elements for the octahedral group O and the 60 element icosahedral group I. The polyhedral groups T, O, and I describe the symmetries of the five Platonic solids [23].

The octahedron and the cube have the same symmetry group and are dual to each other under the S^4 group. The icosahedron and the dodecahedron are dual to each other under the A_5 group and the 12-element group T is the tetrahedral group of which the symmetries are inscribed in S^2 and is the A_4 group. The 24 element octahedral group is denoted as O and is the set of all symmetries inscribed in S^2, which is also the symmetry group of the cube since the eight faces of the octahedron correspond to the eight vertices of the cube. The relationship of the finite and infinitesimal groups is key to understanding the symmetry relation of particles, matter, force fields or gauge fields and the structural topology of space, i.e., real, complex, and abstract spaces. We now relate the toroidal topology and the cuboctahedron geometry to current particle physics.

The 24 element octahedral group is given as

(56a) $$C[\overline{O}] = U_2 \times \tilde{U}_2 \times U_4$$

which is mappable to the conformal supergravity group SU(2,2/1). We can write this as

(56b) $$C[O] = U_1 \times U_1 \times SU_2 \times SU_3 \times SU_3$$

The U_1 can act as the photon (electromagnetic) gauge invariance group and relates to the rotation group SO_3. The other U_1 scalar is the base for space and time as the compact gauge group of the spin two gravitron. The SU_2 group can be associated with weak interactions and $U_1 \times SU_2$ is the group representation of the electroweak force. The SU_3 groups represent the strong color quark – gluon force or gauge field. [20]

Thus we have a topological picture that relates to the unification of the four force fields in the GUT and supersymmetry models. More exactly, the maximal compact space of $C[O]$ is embedded in S^4 or SU(2, 2/1) which yields the 24 element conformal supergravity group. The icosahedron or Klein group yields the set of permutations for S^4 permutation group associated with $C[O]$. Also in the Georgi and Glashow scheme [24], we can generate SU_5 as a 24 element group related to S^4 embedded in $SU_5 = SU_2 \times SU_3$. The key to this approach is the relationship of the finite groups $C[O]$ and the Lie groups such as the SU_n groups. This picture is put forward in detail by Sirag in his significant advancement of fundamental particle physics [20-22].

The eight (8) fundamental spinor states can be expressed in terms of the Riemann sphere S^2 which defines the relationship of spinors to spacetime. The 8 spinor states correspond to the 8 vertices of a cube. For 8 antistates, Sirag can generate all 16 states of the fermions family for a cube and its mirror image cube. In his work, Sirag utilizes the symmetric four group S^4 which is isomorphic to O, the octahedral group.

As before stated, the cube and octahedron are dual to each other under the symmetry operations of the S^4 group. Also, the tetrahedron is dual to the cube under the A_4 group, and the icosahedron and dodecahedron are dual under the A_5 groups. The cover group $C[O']$, which is the DS^4 group, is the cover group of $C[O]$ and $C[\overline{O}]$. The $C[O]$ group is also denoted SU(2, 2/1) and is the compact representation of the Yang-Mills Bosons and $C[\overline{O}]$ represents the matter fields of the Fermions. The Weyl group is SU(2,2) which is related to SU(2, 2/1), the Penrose twistor [25,26], which represents a vortical rotational complex dimensional space, mappable to the Kaluza-Klein model, which relates the electromagnetic metric to the gravitational metric as a five dimensional space [27,28]. The Penrose twistor is a spin space and is like a double torus without a waist. The U_2 group represents the four real spacetime dimensions and $\tilde{U}_2$ the four imaginary spacetime components forming a complex eight space [29-31]. The twistor algebra of this complex eight space is mappable 1 to 1 with the spinor calculus of the Kaluza-Klein geometry, thus electromagnetism is related to the gravitational spacetime metric [29]. The S^4 and $\overline{S^4}$ groups are 24 element groups, as S^4 can be associated with $C[O]$ and $\overline{S^4}$ with $C[\overline{O}]$. The S^4 group is associated with the 24 dimensions of the Grand Unification Theory, or GUT theory. The conjugate group of $\overline{S^4}$ is associated with $U_2 \times \tilde{U}_2 \times U_4$ or for U_4, which is four copies of U_1. That can be written as $U_1 \times U_1 \times U_1 \times U_1$ where $U_1 \times U_1$ represents a torus, hence U_4 represents a double or dual torus. Both $C[O]$ and $C[\overline{O}]$ relate to the T^4 group, where T^n is the direct product of n copies of U_1, called an n torus, which is always an Abelian group. The T in this context refers to the structure of space and time.

We have demonstrated that the cover group of the cuboctahedron generates the double of the torus $U_1 \times U_1$, and hence we demonstrate that this cover group generates the dual torus, which is $U_1 \times U_1$ cross $U_1 \times U_1$ in the Harameinian topology (see Fig. 1), which is defined as the dual torus space. The hourglass topology is directly formed from the topology of the dual sphere. The relationship of the cuboctahedral groups and the dual torus is a fundamental tenet of the Haramein geometric topology and, as seen here, seems to be fundamental for unification [31].

The key is that the infinitesimal Lorentz transformation is related to the concept of the infinitesimal generators of the Lie algebras. We are dealing with both infinitesimal and finite element systems when we consider torque and Coriolis terms in Einstein's field equations. The Lie groups are, of course, the basis of the GUT unification scheme. The relationship between the torus space U_4 and the subset of the $C[\overline{O}]$ and $C[O]$ spaces is the cuboctahedron. Therefore, the modification of Einstein's field equations with the inclusion of torque and Coriolis terms, yields a group theoretical basis in the U_4 metrical space that forms a possible unification of the gravitational force with the strong, weak, and electromagnetic forces in a unified theory.

CONCLUSION

We have developed an extended form of Einstein's field equations in which we include torque and Coriolis forces, and hence torsion effects. New solutions are found to the extended field equations, which generates a modification of the Kerr-Newman solution we term the Haramein-Rauscher solution. We establish a reference frame in the description of the rotating metric that accommodates the complexities of gyroscopic dynamics – torque and Coriolis forces. This approach may allow us to define the origin of spin in terms of the new torque term in the field equations and better describe the formation and structure of galaxies, supernovas, and other astrophysical systems, their plasma dynamics and electromagnetic fields. We formulate a relationship between gravitational forces with torsional effects and the Grand Unification Theory (GUT). This unification is formulated in terms of the metric of the new form of Einstein's field equations which is a U_4 space and the group theoretical basis of the GUT picture. Hence, gravitational forces with spin-like terms may be related to the strong and electroweak forces, comprising a new unification of the four forces.

ACKNOWLEDGEMENTS

We would like to express our appreciation to our colleagues, Ulrich Winter, Michael Coyle, Robert Gray, and Buckley Lofton.

REFERENCES

1. C.W. Misner, K.S. Thorne, J.A. Wheeler, *Gravitation,* (Freedman and Co. 1973), pp.142, 165.
2. Elie Cartan-Albert Einstein, *Letters on Absolute Parallelism 1929-1932,* Edited by Robert Debever, (Princeton University Press 1979).
3. N. Haramein, "Scaling Law for Organized Matter in the Universe," Bull. Am. Phys. Soc. **AB006** (2001).
4. N. Haramein, "Fundamental Dynamics of Black Hole Physics," Bull. Am. Phys. Soc. **Y6.010** (2002).
5. E.A. Rauscher, "Closed Cosmological Solutions to Einstien's Field Equations," Let. Nuovo Cimento **3**, 661 (1972).
6. E.A. Rauscher, "Speculations on a Schwarzschild Universe," UCB/LBNL, LBL-4353 (1975).
7. E.A. Rauscher and N. Haramein, "Cosmogenesis, Current Cosmology and the Evolution of its Physical Parameters," in progress.
8. E.A. Rauscher, "A Unifying Theory of Fundamental Processes," *Lawrence Berkeley National Laboratory Report* (UCRL-20808 June 1971) and Bull. Am. Phys. Soc. **13**, 1643 (1968).
9. H. Weyl, *Classical Groups: Their Invariants and Representation*, (Princeton University Press, Princeton, NJ 1996).
10. R. Hammond, Phys. Rev. D **26,** 1906 (1982).
11. R. Hammond, Gen Rel. and Grav. **20**, 813 (1988).
12. W. de Sitter, *The Astronomical Aspects of the Theory of Relativity*, (University of California Press 1933).
13. J.L. Synge, *Relativity: The General Theory*, (North – Holland, Amsterdam 1960).
14. C. Fronsdel, "Completion and Embedding of the Schwarzschild Solution," Phys. Rev. **116**, 778 (1959).
15. M.D. Kruskal, "Maximal Extension of Schwarzschild Metric," Phys. Rev. **119**, 1743 (1960).
16. G. Szekeres, "On the Singularities of a Riemannian Manifold," Pub. Math. Debrecen , **7,** 285 (1960).
17. N. Haramein and E.A. Rauscher, "A Consideration of Torsion and Coriolis Effects in Einstein's Field Equations," Bull. Am. Phys. Soc. **S10.016** (2003).
18. Robert G. Abraham *et al.*, "The Gemini Deep Deep Survey.I. Introduction to the Survey," Catalogs, and Composite Spectra, AJ. **127**, 2455.
19. R.W. Lindquist and J.A. Wheeler, "Dynamics of a Lattice Universe by the Schwarzschild-Cell Method," Rev. of Mod. Phys., **29**, 432 (1957).
20. S.P. Sirag, International Journal of Theoretical Phys. **22**, 1067 (1983).
21. S.P. Sirag, Bull. Am. Phys. Soc. **27**, 31 (1982).
22. S.P. Sirag, Bull. Am. Phys. Soc. **34**, 1 (1989).
23. H.S.M. Coxeter, *Regular Polytopes*, (Macmillian, New York 1963), 3rd Ed. (Dover, New York 1973).
24. H. Georgi and S.L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).
25. R. Geroch, A. Held and R. Penrose, J. Math. Phys. **14**, 874 (1973).
26. R. Penrose, "The Geometry of the Universe," in *Mathematics Today*, ed. L.A. Steen, (Springer-Verlag 1978).

27. Th. Kaluza, Sitz. Berlin Press, Akad. Wiss. 966 (1921).
28. O. Klein, Z. Phys. **37**, 895 (1926).
29. C. Ramon and E.A. Rauscher, “Superluminal Transformations in Complex Minkowski Spaces,” Found. of Phys. **10**, 661 (1980).
30. E.A. Rauscher, Bull. Am. Phys. Soc. **23**, 84 (1978).
31. N. Haramein, “The Role of the Vacuum Structure on a Revised Bootstrap Model of the GUT Scheme,” Bull. Am. Phys. Soc. **N17.006** (2002).